CLASSICAL AND MODERN IN HEAT TRANSFER

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Abstract

This lecture discusses historically the development of our understanding of internal heat generation in fluid streams and its effect on the temperature and velocity fields in fluids with large Prandtl numbers - oils and polymers - as well as in gases at high velocities. Cross transport of energy in unsteady gas flow is also included.

1. INTRODUCTION

Leafing through some Journals of Heat Transfer, one finds that the lists of references for the papers generally reach back 10 to 20 years. This really is a short period compared with the time span of about 200 years which, it took to develop a heat transfer science.

It should, therefore, be of interest to trace its development from the origin and I will attempt to do that in this lecture for a process with which I am familiar - heat transfer under the influence of internal heat sources.

2. VISCOUS HEATING

In the years between 1880 and 1890, Beauchamp Tower carried out a series of extensive experiments on the lubrication of journal bearings. His work was supported by the Institute of Mechanical Engineers and the results were published in a series of papers. Previous measurements of the frictional resistance of oil bearings had not lead to any consistent results and this was also initially observed by Tower. Then he found, however, that under certain conditions the two load-bearing surfaces were separated by a continuous oil film and that now the results of the measurement were repeatable and consistent. This condition is called hydrodynamic lubrication. Towers experiments found strong interest in the scientific community. Lord Rayleigh commented on them in 1884 at a meeting at Montreal and G. A. Stokes suggested in a letter that the bearing could act as a pump when the journal arranged itself excentrically in the shell, but Towers paper encountered criticism among practicing engineers. A letter published in the Engineer in February 1884 states:

> "as far as any practical purpose was served, the research described in Tower's first report might just as well never have been undertaken. To some minds the fact that the friction between oiled surfaces is reduced might prove valuable, but to the great body of engineers the statement was simply useless. That the frictional coefficient varied with the speed was a matter of no importance, because the fact could not usefully be applied. Nothing was gained by the discovery that the use of an oil bath diminished friction enormously, because in practice oil baths could not be used. So-called scientific research, (he concluded), was rapidly becoming nothing but a method by which considerable incomes might be earned in finding out things of no earthly use to any mortal."

Osborn Reynolds, however, became intensely interested in Tower's experiments and was stimulated to develop his famous hydrodynamic theory of lubrication which he published in the **Philosophical Transactions of the Royal Society** in (Reynolds 1886) . The extensive paper, covering 77 pages, describes at first the physical process in a bearing and concludes that the journal arranges itself as sketched in Figure 1. He then developed a complete mathematical theory of the hydrodynamic process based on the Navier-Stokes equations, which he simplified for the condition of a very thin film.

An integration of the equations by series development results in the solution to the problem. It indicates that, for a particular journal, the thickness of the oil film is essentially constant and that for constant viscosity the resistance should increase proportionately with speed. This, however, disagrees with Tower's findings and Reynolds suspected that internal friction must generate a temperature increase in the film which, through the temperature dependence of viscosity will affect the hydrodynamic process in spite of the fact that the oil film is extremely thin (of order 0.1 mm). He proceeded to measure the viscosity μ of olive oil and found that its dependence on temperature can be approximated by the equation

$$\mu = \mu_0 e^{\beta(T-T_0)} \qquad (1)$$

β is characteristic for the oil (0.3265 kg/ms for olive oil between 16°C and 49°C). Introducing this expression into his equations in an approximate way, he was able to bring his calculated results into complete agreement with Tower's experimental findings.

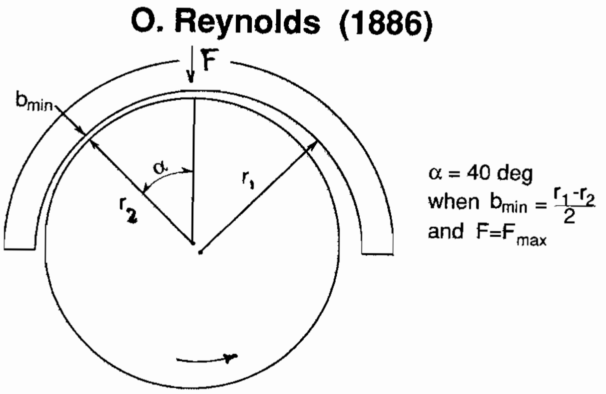

Fig 1) Sketch of a bearing as studied by B. Tower. Equations according to O. Reynolds.

Refinements were added to the hydrodynamic theory in the following decades and only recently have tribologists started to include interfacial effects explaining them by molecular considerations (Beerbower 1985).

From a heat transfer viewpoint, the process discussed above is determined by the balance between the heat which internal friction generates in the oil film and the heat which is conducted to the walls of the bearing. The ratio of the two fluxes, formed with prescribed quantities (U journal speed, k thermal conductivity, T_0 temperature of the bearing surfaces, μ_0 viscosity at T_0) is, therefore, a parameter determining, for instance, the highest temperature T_{max} in the fluid

$$\frac{\mu_0 U^2}{k(T_{max}-T_0)} \qquad (2)$$

Frictional heating is also an important consideration in polymer processing and eqn. (2) has been given the name Brinckman number Br by workers in this field (Pearson 1978).

The parameter β, which characterizes the temperature dependence of the oil, is also determining the velocity and temperature field in the film. A dimensionless parameter describing its influence is

$$\frac{\mu_0 \beta U^2}{k} \qquad (3)$$

This parameter is called Nahme number Na (Pearson 1978).. One expects, therefore, the temperature increase in the oil film to be described by an equation of the form

$$\frac{\mu_0 U_0^2}{k(T_{max}-T_0)} = f\left[\frac{\mu_0 \beta U^2}{k}, \frac{\rho U b}{\mu_0}, \frac{\mu c}{k}\right] \qquad (4)$$

or $\qquad \mathrm{Br} = f(\mathrm{Na}, \mathrm{Re}, \mathrm{Pr}) \qquad (5)$

The term $b = r_1 - r_2$ denotes the clearance of the journal, c the specific heat, and ρ the density of the oil. k, c, and ρ are assumed constant.

A first approximation to the temperature field can be obtained by examination of a model shown in Figure 2. The fluid film is contained in the annular space between two concentric cylinders of almost equal diameters. The outer cylinder is at rest whereas the inner cylinder is impulsively started and rotates then with a constant speed U. Both cylinders are maintained at the temperature T_O. The diagram in Figure 2, (Eckert and Faghri 1986) is an example of the results. The dimensionless velocity u/U and the dimensionless temperature $\beta(T-T_0) = Na/Br$ are plotted against the dimensionless distance y/b from the stationary cylinder surface. The dimensionless time $\underline{t}$ is the parameter on the curves. The figure is for a Nahme number 8 and a Prandtl number 100. Oils have Prandtl numbers between 100 and 10000. The Reynolds number does not appear as a parameter for this model and the asymptotic steady state is also independent of Prandtl number. An analytic expression for the velocity and temperature field for the steady state was reported (Nahme 1940). The dimensionless steadystate maximum temperature $\beta(T_{max}-T_O)$ is read on the abscissa to have the value 0.68. For a fluid with constant viscosity μ_O, it has the value $Na_O/8$. For $Na = 8$ the maximum dimensionless temperature, therefore, has the value 1. The variation of the viscosity with temperature is very noticeable. A typical value of $1/\beta$ for oils is 30 K. The temperature increase in the oil film for the conditions of Figure 2 is $T_m-T_O = 20{,}4°C$.

It was recently pointed out in various papers (i.e., Ayeni 1982) that a thermal runaway condition, also referred to as thermal burst, can occur in which the oil temperature increases exponentially. Such a runaway of the temperature happens, for instance, when a constant torque on the journal is prescribed. In practice such boundary conditions should be rare. The order of magnitude of the time during which this runaway occurs should be comparable to the time during which asymptotic steady state is achieved for a process considered in Figure 2. This time is obtained from the dimensionless parameter $kt/(\rho cb^2) \sim 0.4$. With the following values for oil ($\rho = 864$ kg/m^3, $c = 1.96 \times 10^3$ J/kgK, $k = 0.144$ W/mK) and a film thickness $b = 10^{-4}$m, the time period to attain steady state is $t \sim 10^{-1}$ seconds. If runaway occurs then it happens very rapidly unless set by inertia of the moving parts.

R. Nahme (1940),
E.R.G. Eckert and M. Fagri (1986)

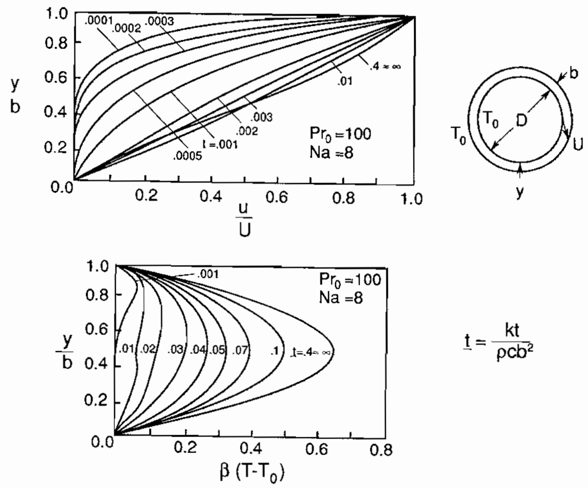

Fig.2) Velocity and temperature profiles in the fluid filling the annulus between an inner rotating and an outer stationary cylinder during the starting period.

Viscous heating has also been studied in polymer processing, for instance, during the extrusion process. Liquid polymers are very viscous and have Prandtl numbers between 10^4 and 10^8 Models for which the viscous heating has been studied by H. H. Winter are shown in Figure 3 (Winter 1977).

3. AERODYNAMIC HEATING

The viscosity of gases is by several orders of magnitude smaller than the viscosity of oils or polymers. Significant temperature increases by internal friction occur, therefore, only at velocities of order of the sound velocity. The speed of airplanes approached such velocities in the years around 1940 and by that time the study of internal heating in high speed air flow became interesting to engineers.

Those concerned with such processes could look back for information to a paper by Ernst Pohlhausen (Pohlhausen 1912). Pohlhausen analysed at the suggestion of Ludwig Prandtl the thermal boundary layer which forms on an adiabatic flat plate exposed to a longitudinal high speed flow

H.H. Winter (1977)

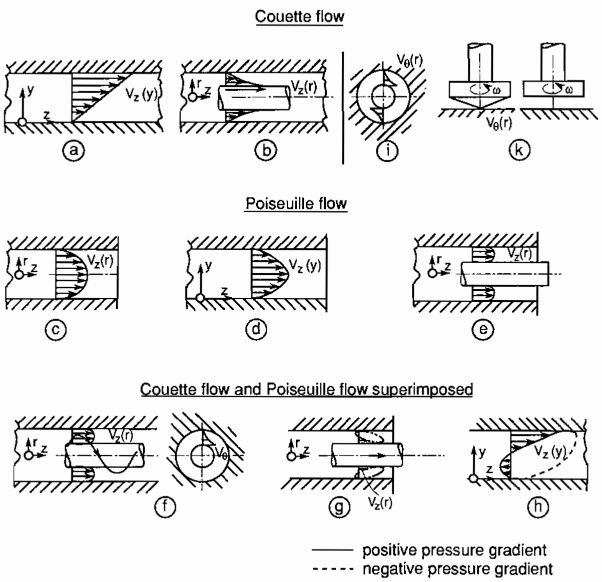

Fig. 3) Various geometries and flow arrangements for which viscous heating was studied. with the velocity U_0 and the temperature T_0 of a fluid with constant properties. The flow is postulated laminar (Figure 4). Internal friction increases the temperature in the boundary layer and raises the plate temperature to the value T_r. This temperature is called today recovery temperature. The results of Pohlhausen's analysis can be well approximated by the equation

$$\frac{T_r-T_o}{U_o^2/(2c_p)} = r = \sqrt{Pr} \qquad (6)$$

in the Prandtl number range from 0.6 to 15. The dimensionless parameter on the left side of the equation is called recovery factor r. The analytical result was verified by Edmond Brun (Brun and Vernotte 1932). He attached a small plate instrumented with the thermocouples to the end of the rotating arm and measured thus the temperature increase of the plate. E. Eckert and W. Weise established by a different method (Eckert and Weise 1940) that the recovery temperature in air increases when the flow in the boundary layer becomes unstable and settles in the turbulent regime to a value well approximated by the equation

$$r = \sqrt[3]{Pr} \qquad (7)$$

E. Pohlhausen (1912)

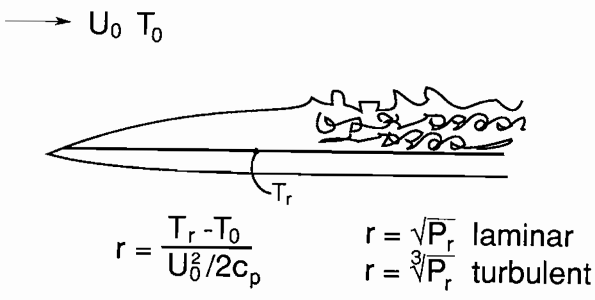

Fig. 4) Highspeed flow over a flat plate forming a laminar and turbulent boundary layer and the resulting recovery factor r.

The aerodynamic heating effect became larger as the speed of aircraft increased and the temperature dependence of the transport properties could not be neglected any more. A simple adjustment, however, could take care of this effect for gases (Eckert 1956). The recovery factor as well as the heat transfer coefficient α_i are defined in terms of enthalpy h instead of temperature T. q is the heat flux at the surface exposed to high speed

$$r = \frac{h_r-h_o}{U_o^2/2},\ q = \alpha_i(h_s-h_r) \qquad (8)$$

gas flow, when the surface (skin of the aircraft) assumes the temperature T_s because it is heated or cooled by radiation or internally. The properties have to be introduced into eqns. (6, 7, 8) at a properly defined reference enthalpy h*.

Temperatures soared to values of order 10000 K in the boundary layer of orbiting or space vehicles which causes the air to dissociate or even to ionize. The consequence is an extremely large variation of the specific heat of air (Figure 5). It turned out, that this can still be accounted for with reasonable approximation by the reference enthalpy method described above.

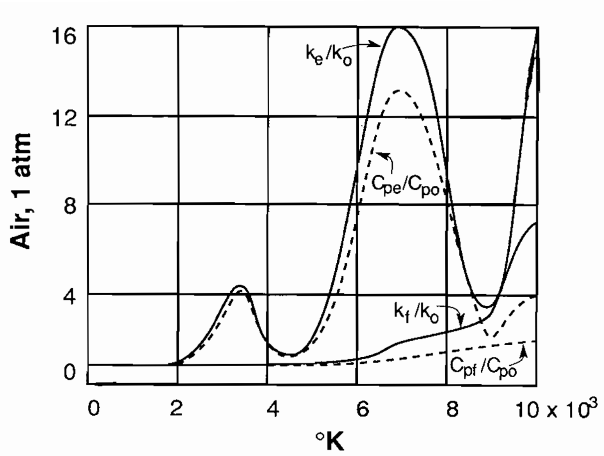

Fig. 5) Thermal conductivity k and specific heat c_p for dissociated and ionized air. The subscript e denotes thermal equilibrium, f denotes frozen state. o indicates a reference state. (From F. C. Hanson, NACA Tech. Note 4150, 1958).

All of the discussions up to this point were extensions of Pohlhausen's analysis for a flat plate and are restricted to flow for which the pressure is uniform in the whole field.

A new effect enters when the pressure varies because, in a compressible fluid, internal energy can be converted into kinetic energy by expansion or vice versa by compression. The ratio of the two modes of energy, therefore, enters as a new parameter into dimensionless relations describing the velocity and temperature field as well as heat transfer. In engineering science it is preferred to use enthalpy instead of internal energy. The dimensionless parameter expressing the ratio of kinetic to internal enthalpy called Eckert number then has the form

$$Ec = \frac{U_o^2}{\Delta h} \qquad (9)$$

(U_0 characteristic velocity, Δh characteristic enthalpy difference) or with constant specific heat $Ec = U_0^2/(c_p \Delta T)$. This parameter can also replace the Brinkmann number through the relation $Br = Ec\,Pr$ to express the ratio of heat generation by friction to heat flux by conduction.

4. TEMPERATURE MEASUREMENTS IN FLUID STREAMS

William Thomson, professor of natural philosophy at the University of Glasgow, Scotland, and J. P. Joule looked in 1883 at the problem of temperature measurement in fluid streams as a sideline when they studied the Thomson-Joule effect (Thomson and Joule 1853). Lloyd Ryan illustrated the results of these studies in a figure reproduced here as Figure 6 (Ryan 1951). A jet of air is released through a small orifice in a thin plate at a pressure differenceΔp. The temperature difference ΔT on the ordinate is the temperature measured by a thermometer exposed to the air jet minus the total temperature of the air upstream of the orifice. The lowest curve was measured when the thermometer was bare. The negative ΔT corresponds to a recovery factor of aerodynamic heating smaller than one. The uppermost curve was obtained when the thermometer was placed inside a rubber hose. Large positive ΔT was later measured by Herbert Sprenger in resonance tubes (Sprenger 1954). The intermediate curve with small negative ΔT was obtained in a test setup in which air was throttled down to a lower pressure by passing a cotton plug (Thomson Joule effect).

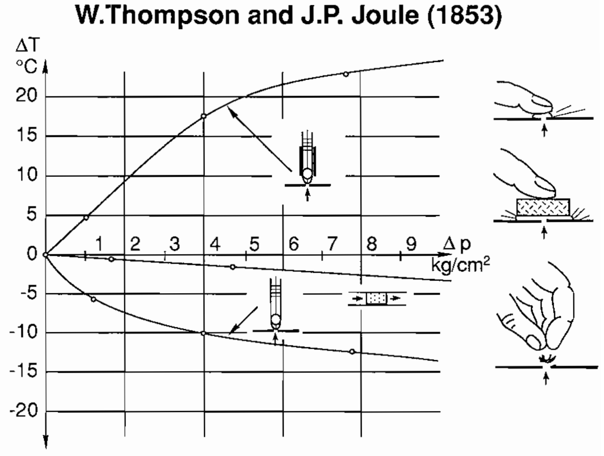

Fig. 6) Temperature increase ΔT by aerodynamic heating for various flow arrangements. Δp denotes the pressure drop across an orifice or a cotton plug. It was changed by holding a finger toward the orifice.

Joule made another effort to measure the temperature in an air stream in 1857 by an experiment which he described in a letter dated March 14, 1857 (Cardwell 1983).

> "It has been blowing a hard gale of wind all day with occasional gusts and lulls. As there was at the same time an overcast sky I thought it would be a good opportunity to try the effect on a thermometer. I first used one of my older ones, with a 5/8 inch diameter bulb. I placed this on a stand on a ledge of the house. By elevating it six inches it became fully exposed to the wind but when lowered the wind did not blow so strongly on the bulb. In the former situation the temperature was 1/3 of a degree Fahrenheit lower than in the latter, the difference in temperature indicating 5 divisions of the scale. I was proceeding further but towards noon the wind got so violent that I was fearful of breaking the thermometer or even of being blown off myself, so I waited till the afternoon when at 2 o.c. the wind had considerably abated."

We know today that, qualitatively, the difference in the indication of the two thermometers is in the right direction but the wind velocities were too small to produce a reasonable temperature difference.

All these measurements appear to have been forgotten for many years. Velocities of order of 100 m/s occurred toward the end of the nineteenth century in steam turbines and they caused Aurel Stodola, at the time the undisputed authority in the field of steam turbines, to make the following remark in his book *Steam and Gas Turbines* (Stodola 1945).

> "If the temperature at a certain point, say, at the mouth of the nozzle, is known, then in the case of saturated steam, the pressure is thereby determined, and the velocity can be obtained as in a. For superheated steam, equations (1) and (2) and the condition equation should be treated in the same manner. The procedure, however, is purposeless, because **no method is known that will determine the true temperature in flowing fluids.** The friction of the fluid against the thermometer or thermocouple generates heat affecting the readings to an extent as yet unknown."

A similar opinion was expressed by W. Nusselt. What Stodola called "true temperature" is the value which could be measured by a sensor moving along with the fluid and it is the temperature which determines the local state of the fluid (assuming local thermodynamic equilibrium). Practically it is difficult to measure that way. There can, however, be defined, in compressible fluids, a different temperature which is easier to measure.

Danielle Bernoulli derived in 1738 for steady flow of an inviscid, incompressible fluid the following equation

$$\frac{p}{\rho} + \frac{U^2}{2} = \frac{p^o}{\rho} \qquad (10)$$

which is written here per unit mass. It can be interpreted as expressing conservation of mechanical (pressure + kinetic) energy and can be used to determine the velocity of a viscous fluid at high Reynolds number by measurement of the total pressure p^o and the static pressure p because these pressures are transmitted to the sensor through the thin viscous boundary layer without change.

In a compressible inviscid fluid, mechanical energy can be exchanged with internal energy by compression or expansion and the equation expressing conservation of energy for an inviscid compressible fluid in steady flow reads now

$$h + \frac{U^2}{2} = h^o \qquad (11)$$

with h denoting the true or static enthalpy (internal + pressure energy) and h^o the total enthalpy. The total enthalpy can be measured by adiabatically slowing down the gas to a velocity zero at the location where information is desired. An additional measurement of the velocity results in the true enthalpy and the state equation provides the true temperature. The same procedure can be used in a real gas as long as the slow down process is adiabatic. A sensor based on this principle was to my knowledge for the first

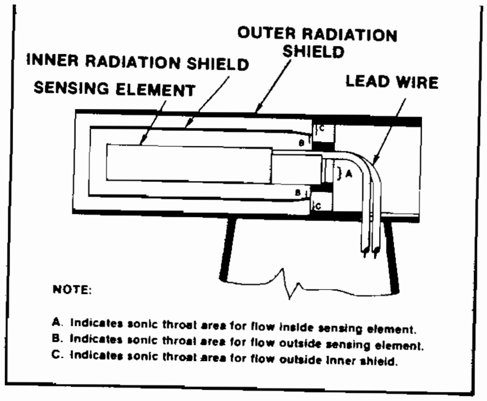

Fig. 7) Total temperature sensor for supersonic flow of air. Copyright Rosemount Inc., 1981. Reprinted by permission.

time described (Franz 1938). A modern version developed by Rosemount Inc., Minneapolis, Minnesota, USA for supersonic velocities is shown in Figure 7. The gas is slowed down in front of the sensor and this occurs adiabatically at high Reynolds numbers. It would, however, lose heat to the walls of the sensor when it would be decelerated to a velocity zero. The sonic throat A provides a small throughflow and optimization by balancing the heat loss against the remaining kinetic energy caused the measured enthalpy increase to approach the desired h^o- h by 99.5%. Two radiation shields make the instrument suitable for measurements in a high temperature environment.

No conversion of mechanical into internal energy occurs in an inviscid incompressible fluid and a total temperature cannot be measured. The analysis of Pohlhausen points to a way to obtain the true temperature in such a fluid by measuring the recovery temperature T_r of a sensor in the shape of a small flat plate. A measurement of the velocity and use of eqn.(6) results in the true temperature T_0. One has to make sure that the flow is laminar and that the Reynolds number is large producing a thin boundary layer. This latter requirement can usually not be fulfilled for measurements in oils or polymers where the sensor has usually to be small. Thus Reynolds numbers are of order 1 to 10^{-4} and recovery factors for such a sensor have to be obtained by a solution of the complete Navier-Stokes equations. A solution of these equations has been reported (Ecket and Shadid 1989). Figure 8 presents the temperature field around an adiabatic cylinder in longitudinal flow at Re = 10^{-2} and Pr = 5×10^7. It can be observed that the temperature field in the fluid is concentrated in a narrow strip around the cylinder in spite of the fact that the character of the flow is "creeping". The lines of constant velocity are nearly parallel to the cylinder surface. This causes the surface temperature of the cylinder (the recovery temperature) to increase in flow direction as opposed to the situation in a boundary layer where the recovery temperature is constant along the flat plate.

5. CROSS TRANSPORT OF ENERGY IN FLUID STREAMS

Most of the methods to calculate flow and heat transfer in engineering devices are for steady flow in spite of the fact that most of the flows with which engineers are concerned are unsteady, i.e., all turbulent flows. Some effects occurring in unsteady flows are missed in this way. One of them is cross transport of energy which is related to aerodynamic heating. It was for the first time observed (Eckert and Weise 1940) when very low temperatures were measured in the separated flow region of an adiabatic cylinder

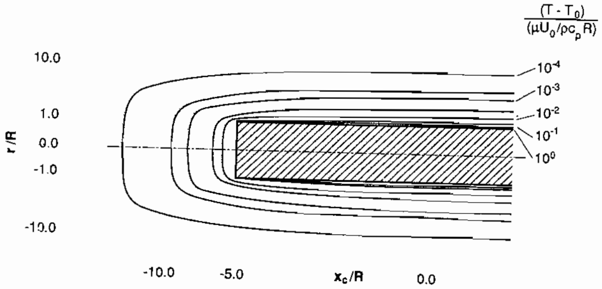

Fig. 8) Temperature field around an unheated cylinder in longitudinal low Re flow of a viscous fluid (large Pr). T local temperature, T_0 upstream temperature, R cylinder radius, r radial distance , x_c distance from cylinder midplane.

E.R.G. Eckert and W. Weise (1940)
Ryan (1951)

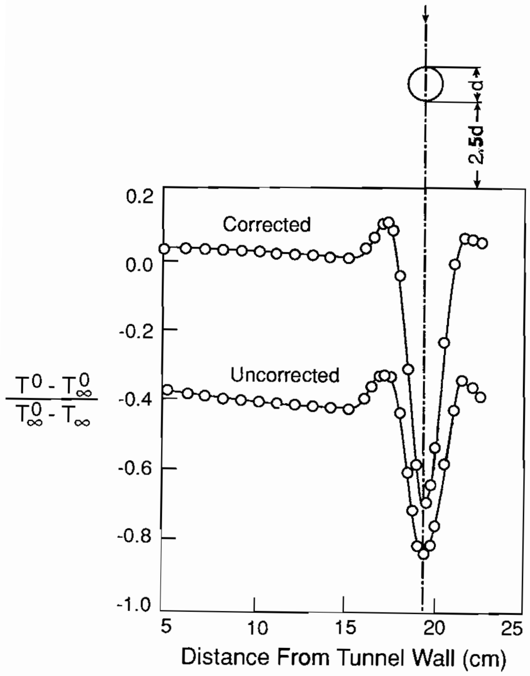

Fig. 9) Time averaged total temperature profile across the wake downstream of a cylinder in air flow normal to its axis. The corrected curve is obtained from a calibration of the thermocouple traversing the wake. To local temperature,

exposed to high velocity subsonic air flow at Reynolds numbers around 10^5 and Mach numbers between 0.4 and 0.9. In some cases, the temperature at the downstream stagnaation line was lower than the static upstream temperature resulting in a negative recovery factor so that the process should be called "aerodynamic cooling". (Ryan 1951) He traversed the wake behind the cylinder with a thermocouple and measured in this way the time averaged total temperature T^O across the wake. Figure 9 presents the results and one observes that T° is low at the center and, to compensate for this, higher than the free stream total temperature T°_∞ at the rims of the wake. The process of this cross transport of energy will be explained with the help of Figure 10. It occurs already in inviscid fluids. In steady flow, the streamlines forming the stream tube in the figure do not move in the direction normal to them. Pressure forces acting on the walls of the stream-tube do, therefore, not perform work and the total enthalpy stays constant along the stream tube. In unsteady flow, the paths traced by the fluid particles are called pathlines and a path tube can be formed with such pathlines. The sketch in Figure 10 may now present such a path tube. In unsteady flow, these path tubes move or fluctuate in a normal direction and in this way, pressure forces can perform work and energy is transferred out or into the path tube, when the time averaged pathlines are curved. In this way, the total enthalpy can change as the fluid moves along the path tube. This is also shown by the unsteady energy equation of an inviscid fluid which can be written in the form

$$\frac{Dh^o}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} \qquad (12)$$

Such cross transport of energy will occur wherever the flow is unsteady and the pathlines are curved - a situation quite frequent in engineering devices (Eckert 1987). It was recently evidenced by the fact that local time averaged energy balances of the flow through compressor or turbine stages

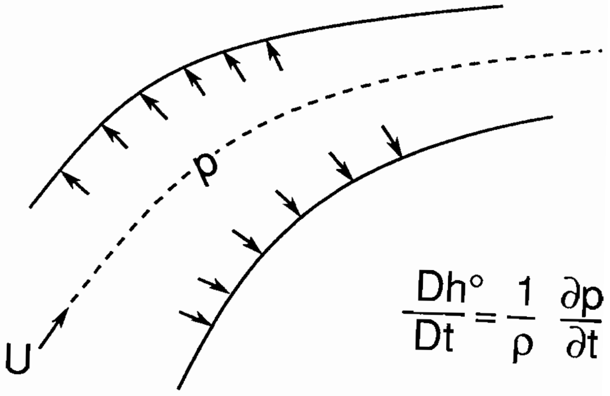

Fig. 10) The action of pressure p on the walls of a stream tube in steady flow (or a path tube in unsteady flow). The equation relates the change of total enthalpy h° along the path tube to the local pressure change.

lead to thermodynamic efficiencies with values larger than one at some locations (Weisier, Bauer and Oklishi 1987) Real local balances are thus impossible as long as one can not predict the cross transport of energy quantitatively.

6. CONCLUSION

It is interesting to speculate about what motivated the early pioneers in the field to start a specific research project and what impact the study had.

At the time of Tower and Reynolds, in the years 1880-86, there was certainly a need to prolong the life and to decrease the resistance of bearings. Both of the researchers had the vision to realize that these goals could only be reached by systematic research. The following years proved them right because the development of bearings for high speed engines like steam turbines would not have been possible without the hydrodynamic theory. This is documented by the fact that Kingsbury of Pittsburgh, USA founded thirty years later the Osborn Reynolds Fellowship in Engineering at the University of Manchester, where Osborn had been Professor, with the citation

> "as some recognition of the debt which he owed to Reynolds' researches in lubrication in connection with the development of the Kingsbury thrust bearing"

Aerodynamic heating and temperature measurements in fluid streams are intimately connected and will be looked at together. No need for this knowledge existed in engineering when Thomson and Joule carried out their experiments in 1883 and 1887. They were driven by their curiosity in thermodyamic processes. No need to know the effect of internal friction existed even in 1912 when Ernst Pohlhausen analyzed heat transfer in a laminar boundary layer formed on a flat plate in longitudinal flow. He considered two boundary conditions: 1) heat transfer from the plate surface into the fluid when the plate is heated to a uniform and constant temperature higher than the approach fluid temperature. Heat generation by internal friction in the fluid was neglected. 2) The plate is adiabatic and its surface temperature was calculated under the influence of internal heating by friction in the fluid. Pohlhausen called this the plate thermometer. Only this part of the paper was discussed in the section on aerodynamic heating.

As a mathematician, Pohlhausen does not mention in his paper any application of his problem but Ludwig Prandtl who suggested this study was, of course, aware of the many technological processes in which heat transfer occurs in external flow. He may also have realized the basic problem of temperature measurement in flowing fluids, but velocities in heat transfer devices at that time were too small for aerodynamic heating to have any discernable effect and so the plate thermometer was forgotten as Stodola's comment indicates. Only with the advent of high speed flight, became aerodynamic heating the concern of aeronautical engineers, and it became the main problem during the reentry of missiles and spaceships from space, a problem which had to be solved before such vehicles could be developed.

Aerodynamic cooling was discovered experimentally by Eckert and Weise in 1940, but the process which causes it was not understood. A letter from Ludwig Prandtl stated this and suggested errors in the measurements to have caused the effect. Lloyd Ryan, however, confirmed in 1951 the results and showed that the consequence is a cross transport of energy normal to the mainflow direction in the wake. He also mentioned that Jakob Ackeret suggested as cause of the effect the unsteady, fluctuating character of the flow. Until very recently, aerodynamic cooling remained a problem without an application. Only in recent years did it become clear that cross transport of energy influences the local performance of the fluids in axial compressors and turbines and should be considered in its analysis. Kurosaka (Kurosaka, Graham and Shang 1989) was able to do this for the Kármán vortex street by calculating the instantaneous velocity and temperature fields using a high speed electronic computer. To predict the cross transport for the involved flow pattern in compressor and turbine stages is a task for the future.

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